Non-linear Visco-Resistive Collisional Transport in Toroidal Elliptical Plasmas with Triangularity and Hole Currents

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In this conference we will look for the hole effect in tokamak confinement and diffusion by using non-linear MHD treatment. The main simplification in our analysis is that turbulence is very low, in such a way that vorticity can be neglected. Therefore anomalous transport is not included. In spite of our simplifications, our treatment could be of interest in the H mode, since turbulence and vorticity are very low, because of internal barriers. Viscosity and non-linear flows are included in our analysis in the most simple way.

Whilst vorticity, $\vec{\omega} = \nabla \times \vec{\mathrm{v}}$, may be important within an internal barrier, however outside that barrier, the vorticity which is associated with turbulence, is not so significant and it could be neglected, as we assume in this work.

In a previous work [P. Martín, E. Castro and M. G. Haines, Phys. Plasmas **12**, 102505 (2005)] we show that starting from the steady state MHD momentum equation

$$\rho \, \vec{\mathrm{v}} . \nabla \, \vec{\mathrm{v}} = \frac{1}{\mathrm{c}} \vec{j} \times \vec{B} - \nabla \, p + \rho \, \nu \, \nabla^2 \vec{\mathrm{v}} \quad , \qquad (1)$$

We can arrive to the equation

$$\nabla\left(\frac{\mathrm{v}^2}{2}+\mathrm{W}(\rho,\mathrm{T})-\nu\,\nabla.\vec{\mathrm{v}}\right)=\frac{1}{\mathrm{c}\,\rho}\vec{\mathrm{j}}\times\vec{\mathrm{B}}-\vec{\mathrm{v}}\times\vec{\omega}-\nu\,\nabla\times\vec{\omega}\,, \qquad (2)$$

where

$$\vec{\omega}=\nabla\times\vec{\mathrm{v}} \qquad (3)$$

The pressure is given by

$$\mathrm{p}=\mathrm{S}(\psi)\;\rho^{\gamma}\,, \qquad (4)$$

The entropy $\mathrm{S}(\psi)$ is conserved in each magnetic surface [Guazzotto, Betti, Manickam and Kaye, Phys. Plasmas 11, 604 (2004)]. The enthalpy $W(\rho,T)$ is given by

$$W(\rho,T)=\int^{\rho}\frac{1}{\rho}\frac{\partial p}{\partial\rho}d\rho \qquad , \qquad (5)$$

where the integral is performed with T constant along a magnetic line and ψ is the usual poloidal magnetic flux.

The auxiliary function $F(\vec{\mathrm{v}}, \rho, T, \nu)$ can now be defined as

$$F(\vec{\mathrm{v}}, \rho, \mathrm{T}, \nu) = \frac{\mathrm{v}^2}{2} + W(\rho, T) - \nu \nabla . \vec{\mathrm{v}} \qquad , \qquad (6)$$

and this equation can be written also as

$$\nabla F(\vec{\mathrm{v}}, \rho, \mathrm{T}, \nu) = \frac{1}{c\ \rho} \vec{j} \times \vec{B} + \vec{\mathrm{v}} \times \vec{\omega} - \nu \nabla \times \vec{\omega} \ . \qquad (7)$$

Performing now an expansion in the vorticity parameter $\vec{\omega}$, then the lower terms in equation (7) will be

$$\nabla F(\vec{\mathrm{v}}, \rho, \mathrm{T}, \nu) = \frac{1}{c\ \rho} \vec{j} \times \vec{B} \ , \ \Rightarrow \quad \vec{B} . \nabla F(\vec{\mathrm{v}}, \rho, T, \nu) = 0 \ , \qquad (8)$$

thus $F(\vec{\mathrm{v}}, \rho, T, \nu)$ is constant along a magnetic surface, that is

$$F(\vec{\mathrm{v}}, \rho, \mathrm{T}, \nu) = \widetilde{F}(\psi) = F(\widetilde{\sigma}) \quad ,$$

where $\widetilde{\sigma}$ is the first coordinate of the system of geometrical orthogonal curvilinear coordinates $\widetilde{\sigma}$, $\widetilde{s}$ and φ defined in previous works.

Furthermore, since in a very general way ρ can be taken as

(V. I. Il'gisonov and Yu. I. Pozdnyakov, JETP letters **71**, 314 (2000))

$$\rho(\psi) = \rho_0(\psi) \exp[\omega^2 \gamma^2 / 2T] \quad , \qquad (9)$$

Then in lowest order $\rho(\psi)$ will be $\rho_0(\psi)$ and the main equation becomes

$$\nabla F = \frac{1}{c\ \rho_0} \vec{j} \times \vec{B} \ . \qquad (10)$$

On the other hand, it was proved in a preceding paper than the gradient of $F(\tilde{\sigma})$ can be determined by using the curvatures κ_σ of the family of curves ortogonal to the magnetic surface in each meridian plane, that is,

$$\frac{\partial F(\tilde{\sigma})}{\partial \sigma} = \left(\frac{\partial F(\tilde{\sigma})}{\partial \sigma} \right)_1 \exp[-\int_0^s \kappa_\sigma \, ds^{'}] \quad , \qquad (11)$$

where the subindex 1 denotes the outmost point in each magnetic surface, and we are using a previously described system of coordinates for tokamak (P. Martín, PoP **7**, 2915 (2000).

The previous exponential appears very often in this work, and it is useful to denote in a simplified way by

$$\mu(\tilde{\sigma},\tilde{s}) = \exp[-\int_0^s \kappa_\sigma \, ds^{'}] = \frac{1}{h_\sigma(\tilde{\sigma}, \tilde{s})} \quad , \qquad (12)$$

In order to study diffusion in the SOL surface, the normal diffusion velocity is given by

$$\overline{\mathrm{v}} = \frac{1}{\oint R\,ds} \oint \left(\frac{\vec{E}\times\vec{B}}{B^2} - \eta_{\perp} \frac{\nabla F}{B^2} \right) . (-\hat{n})\, ds \quad , \qquad (13)$$

$$\overline{\mathrm{v}} = -\frac{1}{\oint R\,ds} \oint \left(\eta_{\perp} \frac{R}{B^2} \frac{\partial F}{\partial \sigma} - \frac{\vec{E}_p \vec{B}_{\varphi}}{B^2} + \frac{R E_{\varphi} B_p}{B^2} \right) ds \quad , \qquad (14)$$

where $(-\hat{n})$ is the normal to the magnetic surface ($\hat{n}$ is the main normal of the magnetic field curve obtained by intersection of the magnetic surface with a meridian plane), $\eta_{\perp}$ is the normal resistivity and the integral is performed along a poloidal magnetic field curve.

From the equation

$$\nabla . \vec{j} = \nabla . (j_{||} \hat{B} + \vec{j}_{\perp}) = 0 \quad ,$$

and

$$\nabla F = \frac{1}{c\;\rho_0} \vec{j} \times \vec{B} \quad .$$

It is obtained

$$\frac{\partial}{\partial s}\left(\frac{j_{||}}{B}\right) = -\frac{\rho_0 B_\varphi}{c B_p}\frac{\partial F}{\partial \sigma}\frac{\partial}{\partial s}\left(\frac{1}{B^2}\right), \qquad (15)$$

By a cumbersome manipulation of the MHD equations, it can be proved that

$$\frac{\partial}{\partial s}\left(\frac{\rho B_\varphi}{B_p}\frac{\partial F}{\partial \tilde{\sigma}}\right) = 0 \quad , \qquad (16)$$

Now Eq.(15) can be integrated given

$$j_{||} = -\frac{\rho_0 B_\varphi}{c B B_p}\frac{\partial F}{\partial \sigma} + G(\tilde{\sigma}) B, \qquad (17)$$

where $G(\tilde{\sigma})$ is coming as a constant of integration and it has to be found. $G(\tilde{\sigma})$ is determined by the condition that the electric potential $\phi(\vec{r})$ is univaluated. The procedure is as follows:

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t} = -\nabla\phi - \frac{\partial A_\varphi}{\partial t}\hat{\varphi} \,, \tag{18}$$

$$\vec{E}.\vec{B} = E_\varphi B_\varphi - \vec{B}.\nabla\phi = \eta_{||} j_{||} B \,, \tag{19}$$

$$0 = \oint \frac{\partial\phi}{\partial s} ds = -\oint \eta_{||} \frac{j_{||} B}{B_p} ds + \oint \frac{E_\phi B_\phi}{B_p} ds \,, \tag{20}$$

$$G(\tilde{\sigma}) = \left[\oint \frac{\rho_0 B_\phi}{B_p^2} \frac{\partial F}{\partial\sigma} ds + \frac{1}{\eta_{||}} \oint \frac{E_\phi B_\phi}{B_p} ds \right] \Bigg/ \left(\oint \frac{B^2}{B_p} ds \right). \tag{21}$$

Now E_p can be determined by Ohm law, using the previous result for $\vec{j}_{||}$, that is,

$$E_p = \left(\eta_{||}\, j_{||} B - E_\varphi\, B_\varphi \right) / \, B_p, \tag{22}$$

Using these results, the average normal velocity will be

$$\overline{v} = - \frac{\rho_0\, \eta_\perp}{B^2} \left(\frac{\partial F}{\partial \sigma} \right)_1 \frac{1}{I_0} \left[\frac{\tilde{I}_1}{R_1^2} + \frac{\eta_{||}}{\eta_\perp} \frac{1}{R_1^2\, \tilde{\gamma}_1^2} \left(\tilde{I}_3 - \frac{\tilde{I}_1^2}{\tilde{I}_4} \right) \right] - \frac{E_{\varphi_1}}{B_{\varphi_1} \tilde{\gamma}_1\, \tilde{I}_0} \left[\tilde{I}_7 - \frac{\tilde{I}_1\, \tilde{I}_0}{\tilde{I}_4} + \tilde{\gamma}_1^2\, \tilde{I}_5 \right], \tag{23}$$

where

$$\tilde{\gamma}_1 = \frac{B_{p_1}}{B_{\varphi_1}} \quad ,$$

and :

$$\widetilde{I}_0 = \oint R\,ds \quad ,$$

$$\widetilde{I}_1 = \oint \frac{R\,ds}{\mu(s)} \ ,$$

$$\widetilde{I}_2 = \oint \frac{R^3\ \mu(s)\ ds}{1+\widetilde{\gamma}_1\,\widetilde{\mu}^2(s)} \quad ,$$

$$\widetilde{I}_3 = \oint \frac{R^3\ ds}{1+\widetilde{\gamma}_1\,\widetilde{\mu}(s)} \quad ,$$

$$\widetilde{I}_4 = \oint \frac{(1\ +\ \widetilde{\gamma}_1{}^2\ \ \widetilde{\mu}^2(s))\ ds}{R\ \mu(s)} \quad ,$$

$$\widetilde{I}_5 = \oint \frac{R\ \mu(s)\,ds}{(1\ +\ \widetilde{\gamma}_1{}^2\ \ \widetilde{\mu}^2(s))} \quad ,$$

$$\widetilde{I}_6 = \oint \frac{ds}{R\,\mu(s)} \quad ,$$

$$\widetilde{I}_7 = \oint \frac{R\ \ ds}{\mu(s)\ (1\ +\ \widetilde{\gamma}_1{}^2\ \ \widetilde{\mu}^2(s))} \quad .$$

Dimensionless functions $\tilde{\bar{v}}$ are obtained by defining

$$\tilde{\bar{v}} = \frac{\bar{v}}{v_{oc}} \qquad \text{and :} \qquad \tilde{E}_{\varphi} = \frac{E_{\varphi 1}}{-\dfrac{\rho_0 \eta_\perp}{B_{\varphi 1}} \left(\dfrac{\partial F}{\partial \sigma} \right)_1} ,$$

where the velocity for normalization v_{oc} is the diffusion velocity for the circular case, with $E_{\varphi_1} = 0$, which looks as a general Pfirst-Schlüter velocity.

In the case of circular cross sections, after some simplification the Pfirst-Schlüter result is recovered. All the previous integrals can also be written in a dimensionless way, which is not written down for economy.

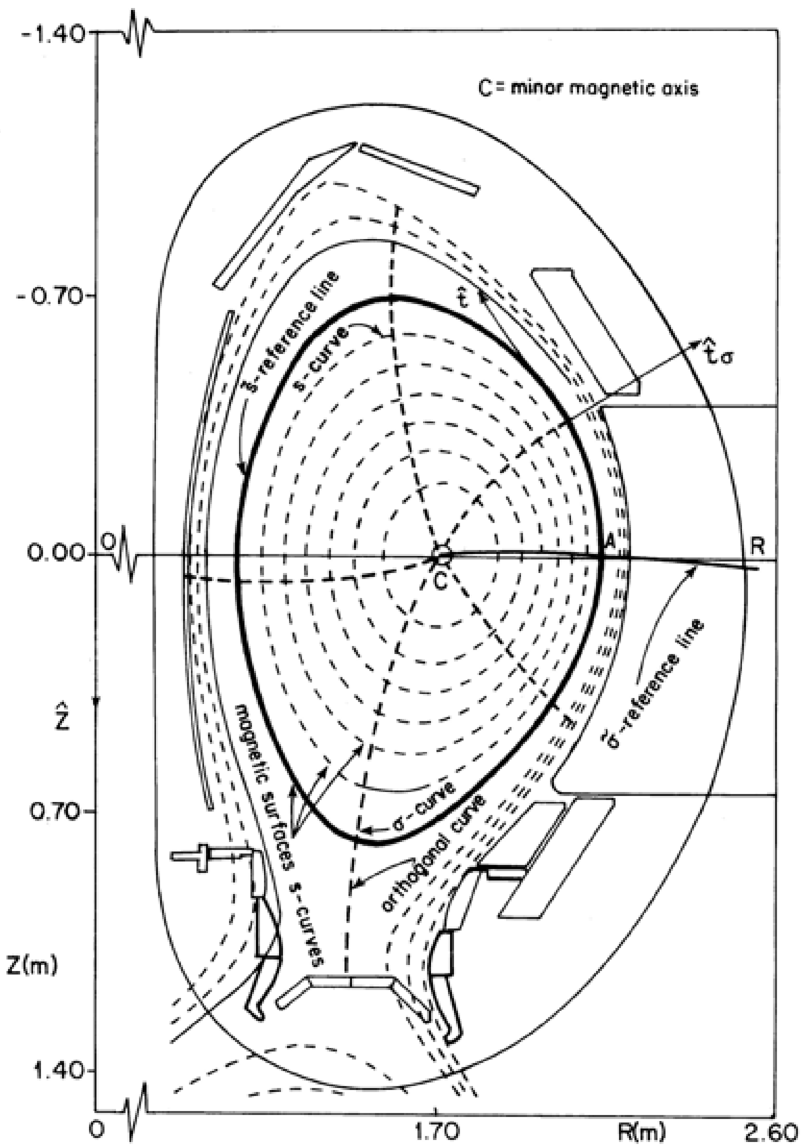


Figure 1: Cross section of the tokamak magnetic surface showing the reference curves for the coordinates used in the text.

Two family of magnetic field curves are analyzed and compared. The first one is for plasmas with central current and the second one is for plasmas with hole current. The equations for the first family are taken from the analysis performed by H. Weitner (PoP-1981) on the Grad-Shafranov equation, considering ellipticity and triangularity,

$$R = R(\lambda,\theta) = R_m + \lambda\, a \cos\theta - \lambda^2 \left[\Delta(a) + \frac{a\,T(a)}{4} \left(1 - \cos 2\theta\right) \right], \qquad (25)$$

$$z = z(\lambda,\theta) = a\,\lambda\, E(a) \left(\sin\theta - \frac{\lambda\, Ta}{4} \sin 2\theta \right) \quad , \qquad (26)$$

In the case of plasmas with hole current, the form of the ellipticity and triangularity are taken from V. Yavorskij et. al., Nuclear Fusion **43**, 1077 (2003), in this way the Grad-Shafranov shift, the ellipticity and triangularity are given by

$$\Delta(\lambda) = \Delta o\,(1-\lambda^2) \quad ,$$

$$\Lambda(\lambda) = \Lambda a\, \lambda^2 \quad ,$$

$$k(\lambda) = ka - (ka - ko)\,(1-\lambda^2)^2 - 0.5\, ka'\, \lambda^2\,(1-\lambda^2) \quad .$$

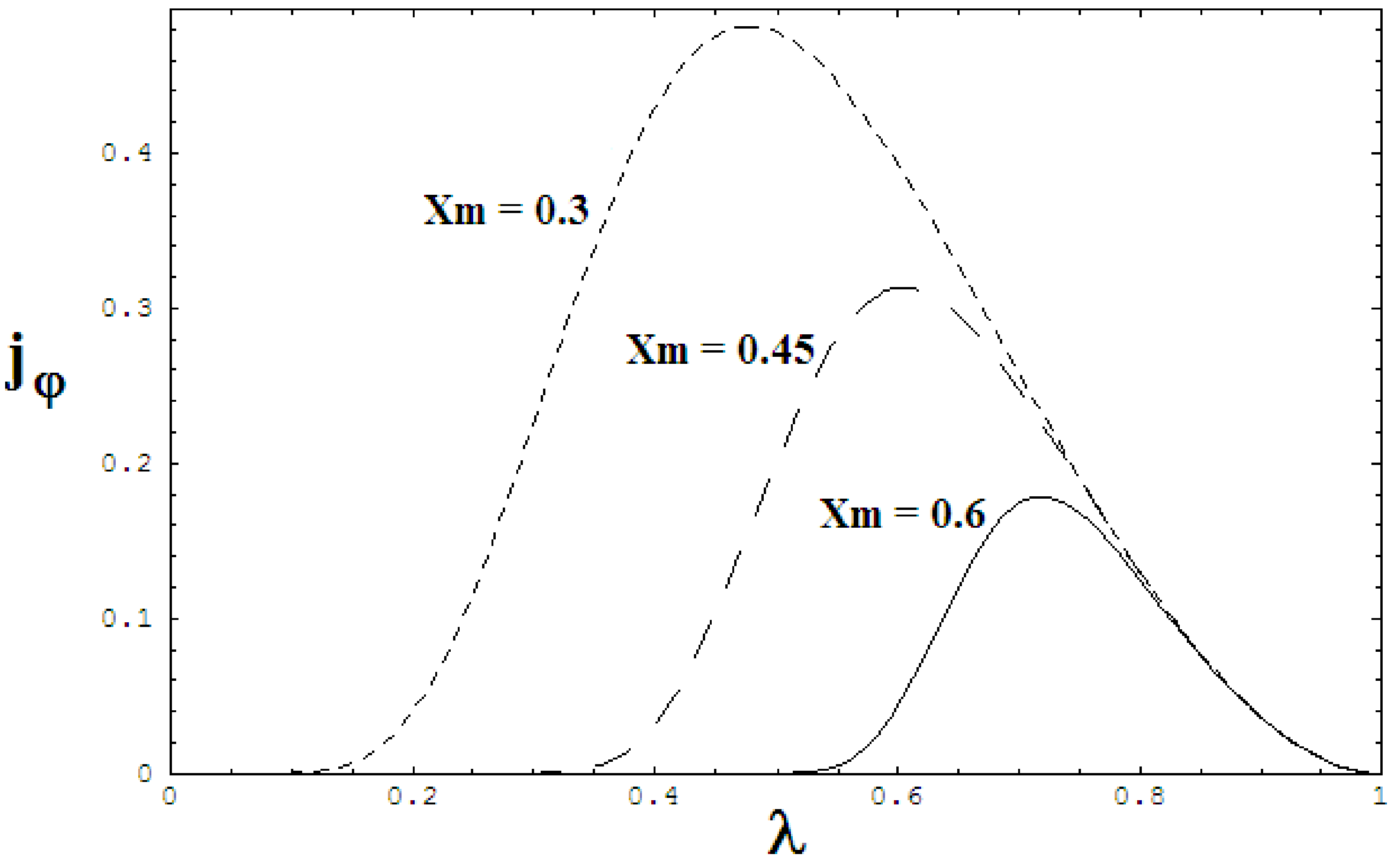


Figure 2

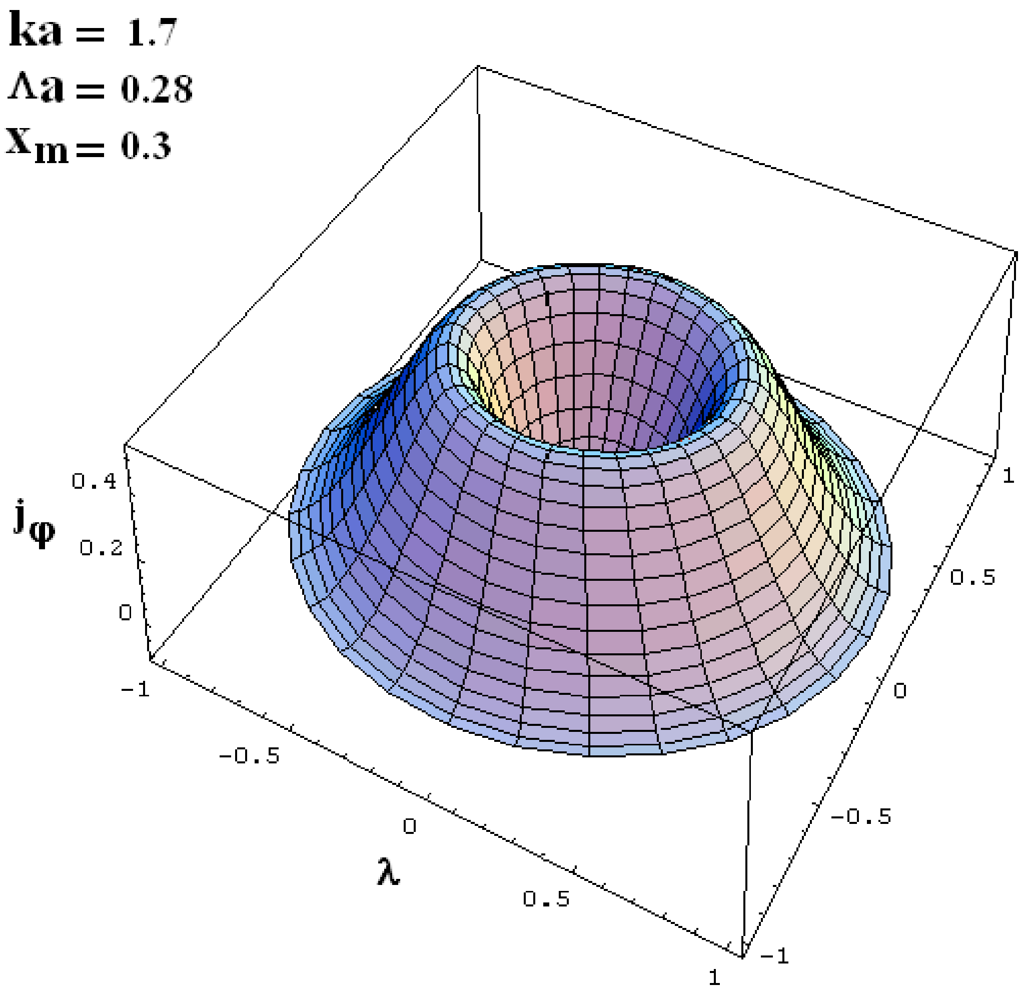


Figure 3

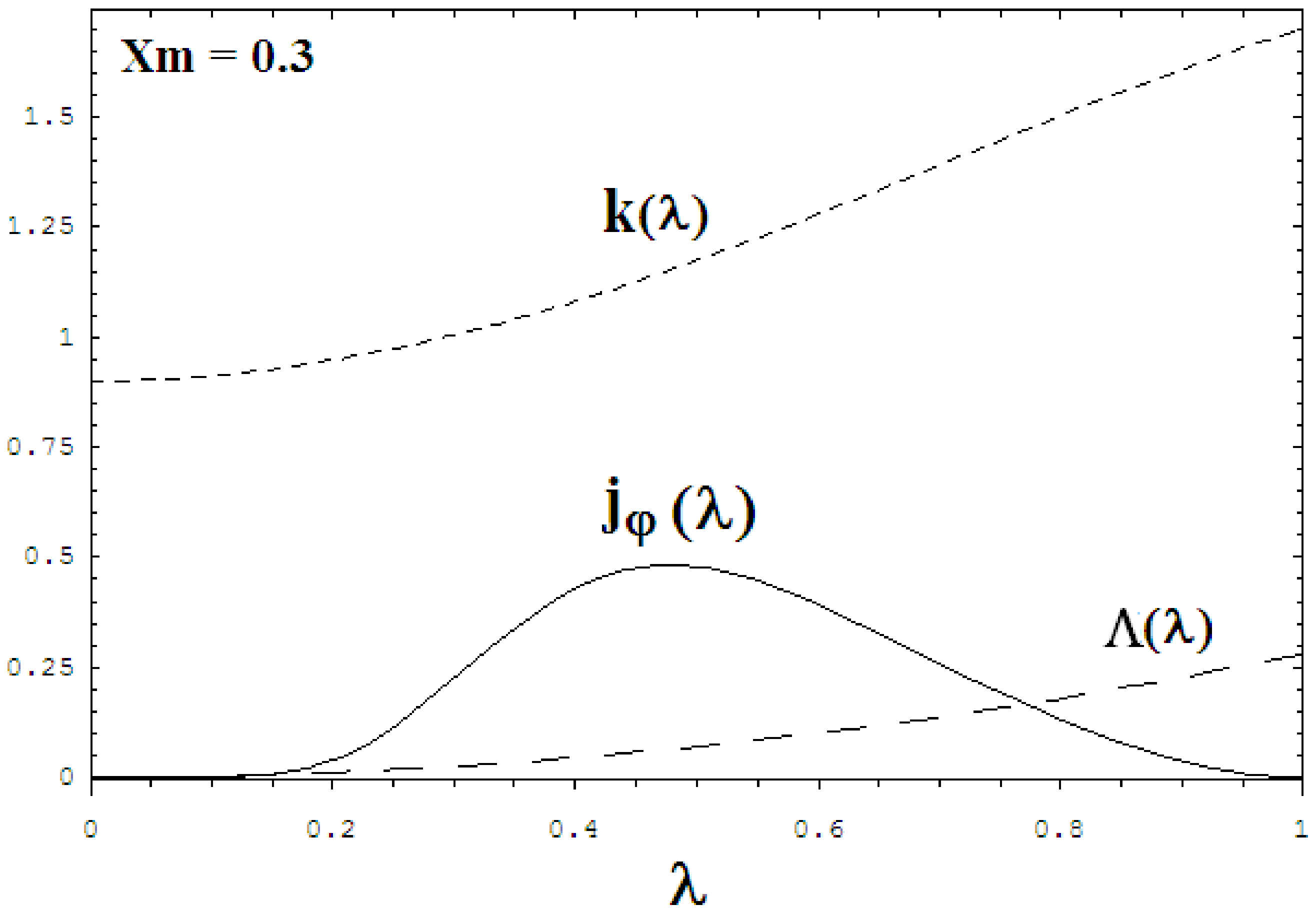


Figure 4

where λ is

$$\lambda = \frac{\tilde{\sigma}}{a} = \frac{r}{a}$$

In that paper, they also introduce a parameter x_m , which is useful to adjust the analytic form to the experimental data, $x_m = 0.6$ for JET, and $x_m = 0.3$ and 0.45 for the JT-60U

$$j_\varphi = (1 - \lambda^2)^2 (1 - (1-\lambda)^6)^{2m} \quad ,$$

$$m = \frac{3((1-x_m)^{-6} - 1)}{(1 + x_m{}^{-1})} \quad ,$$

The computation of the integrals I_i, can now be performed and the results will be shown in the figures.

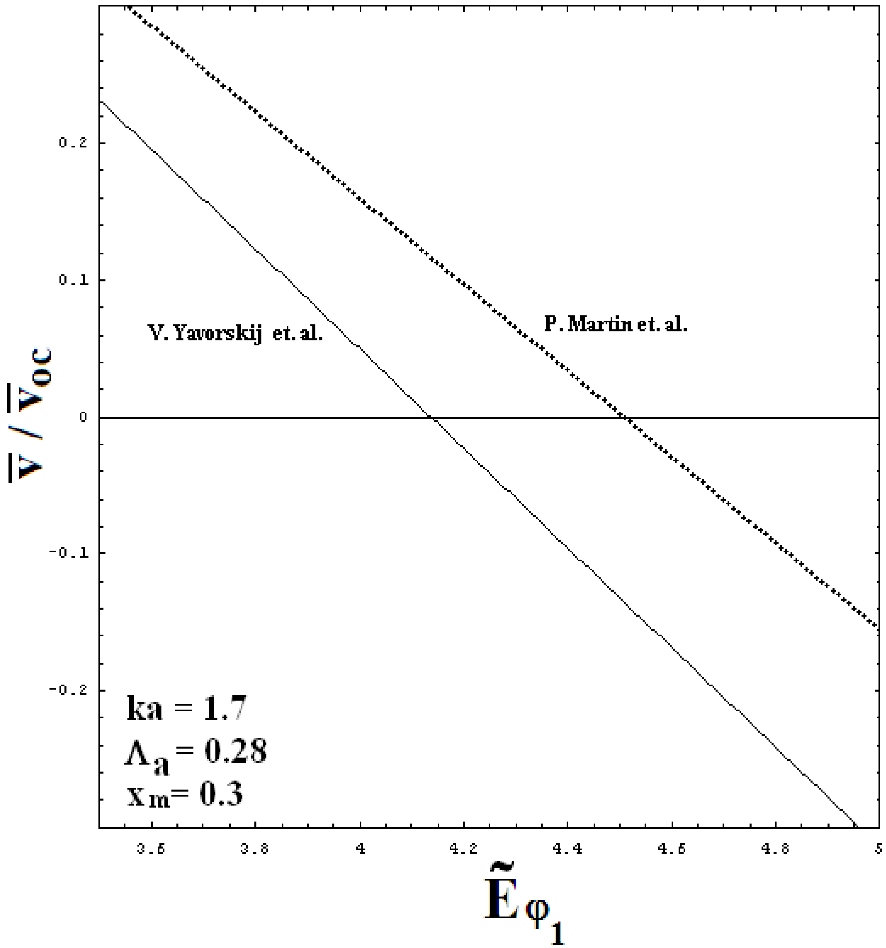
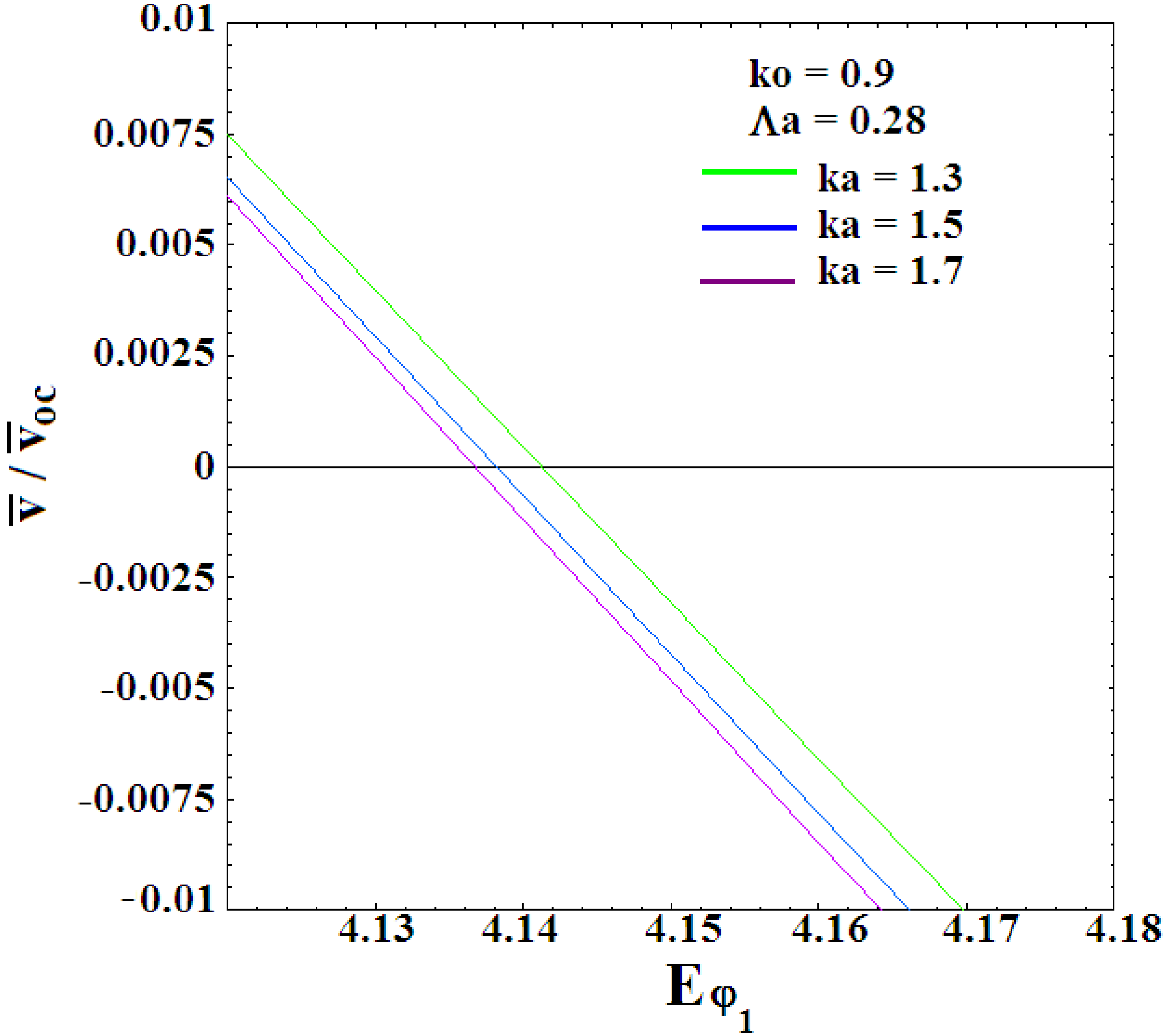


Figure 5

In Figure 5 the dimensionless diffusion velocity versus the dimensionless inductive field are shown for the characteristic parameters in Yavorskij paper

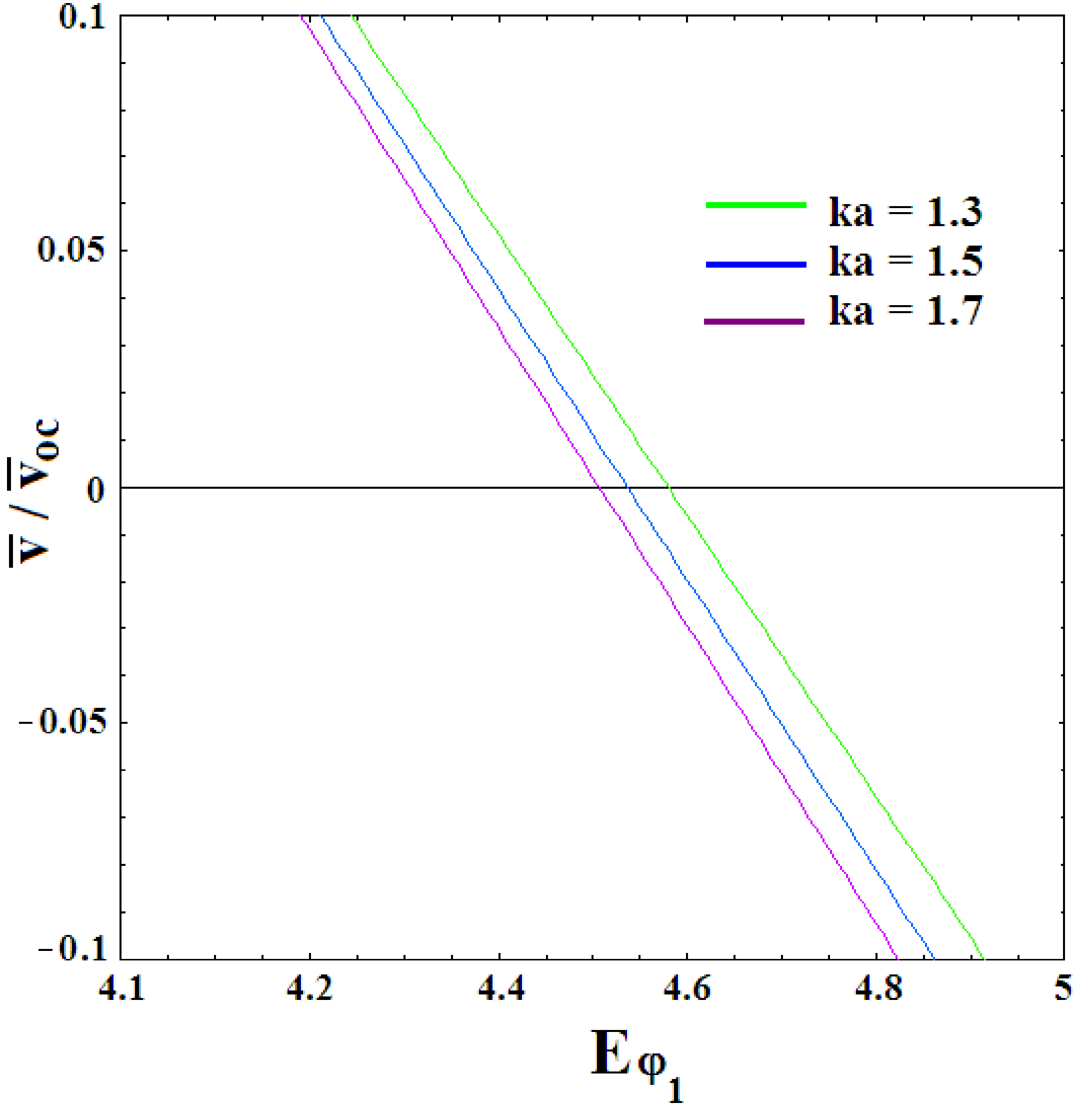
$\Delta(a) = 0.15$, $k(a)=1.7$ and $\Lambda(a)=0.28$

The effect of the hole is evident, there is an increasing in confinement . For some values of $\tilde{E}_{\varphi_1}$, the velocity $\tilde{\tilde{V}}$ will be negative, (which it means confinement), if there is hole current but $\tilde{\tilde{V}}$ will be positive in other case.



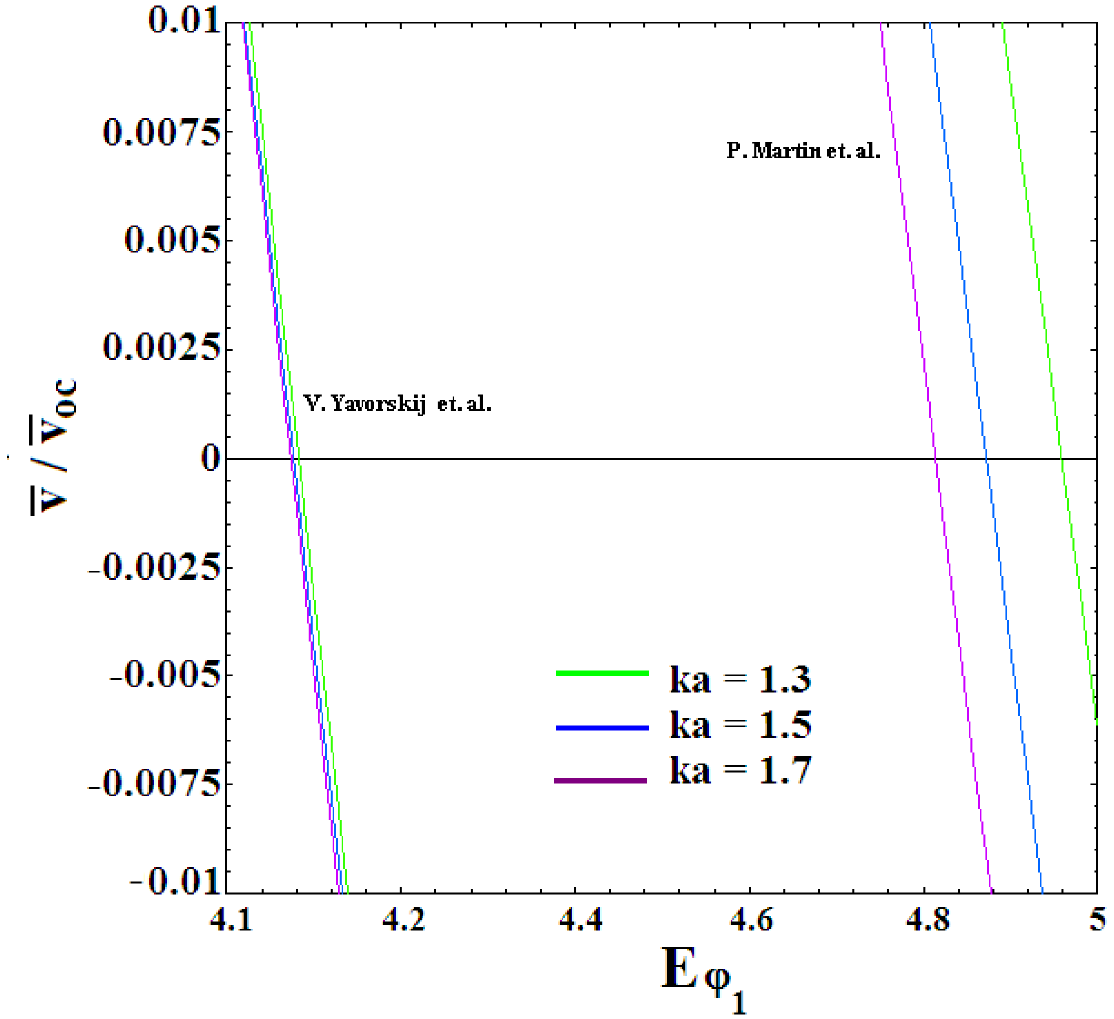
0.01
0.0075
0.005
0.0025
0
-0.0025
-0.005
-0.0075
-0.01
ko = 0.9
Λa = 0.28
ka = 1.3
ka = 1.5
ka = 1.7
4.13
4.14
4.15
4.16
4.17
4.18
$\bar{V} / \bar{V}_{0c}$
E_{φ_1}

Figure 6b



0.1
0.05
0
-0.05
-0.1
$\bar{V} / \bar{V}_{0c}$
4.1
4.2
4.4
4.6
4.8
5
E_{φ_1}
ka = 1.3
ka = 1.5
ka = 1.7

Figure 6a



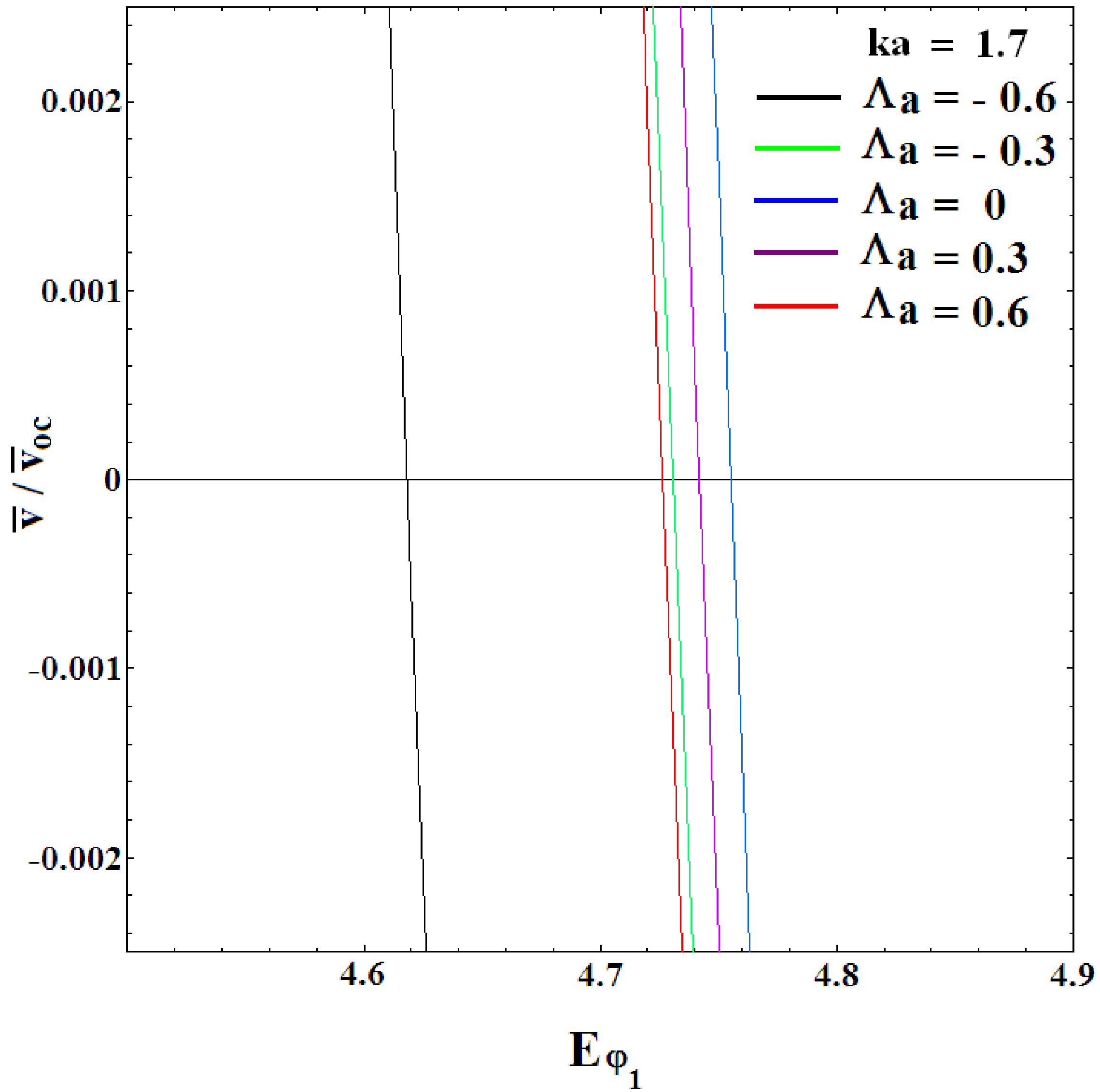
0.01
0.0075
0.005
0.0025
0
-0.0025
-0.005
-0.0075
-0.01
4.1
4.2
4.4
4.6
4.8
5
$\bar{V} / \bar{V}_{0c}$
E_{φ_1}
P. Martin et. al.
V. Yavorskij et. al.
ka = 1.3
ka = 1.5
ka = 1.7

Figure 6c

Figure 6a, shown the same kind of graph for the no-hole case, showing that confinement improves with increasing elongation. The same kind of pattern appears in Figure 6b, where hole case is considered, now similar results are obtained, but with smaller values of $\widetilde{E}_{\varphi_1}$, Figure 6(c) shown both Figures 6a and 6b, in the same graph, for a better comparison.

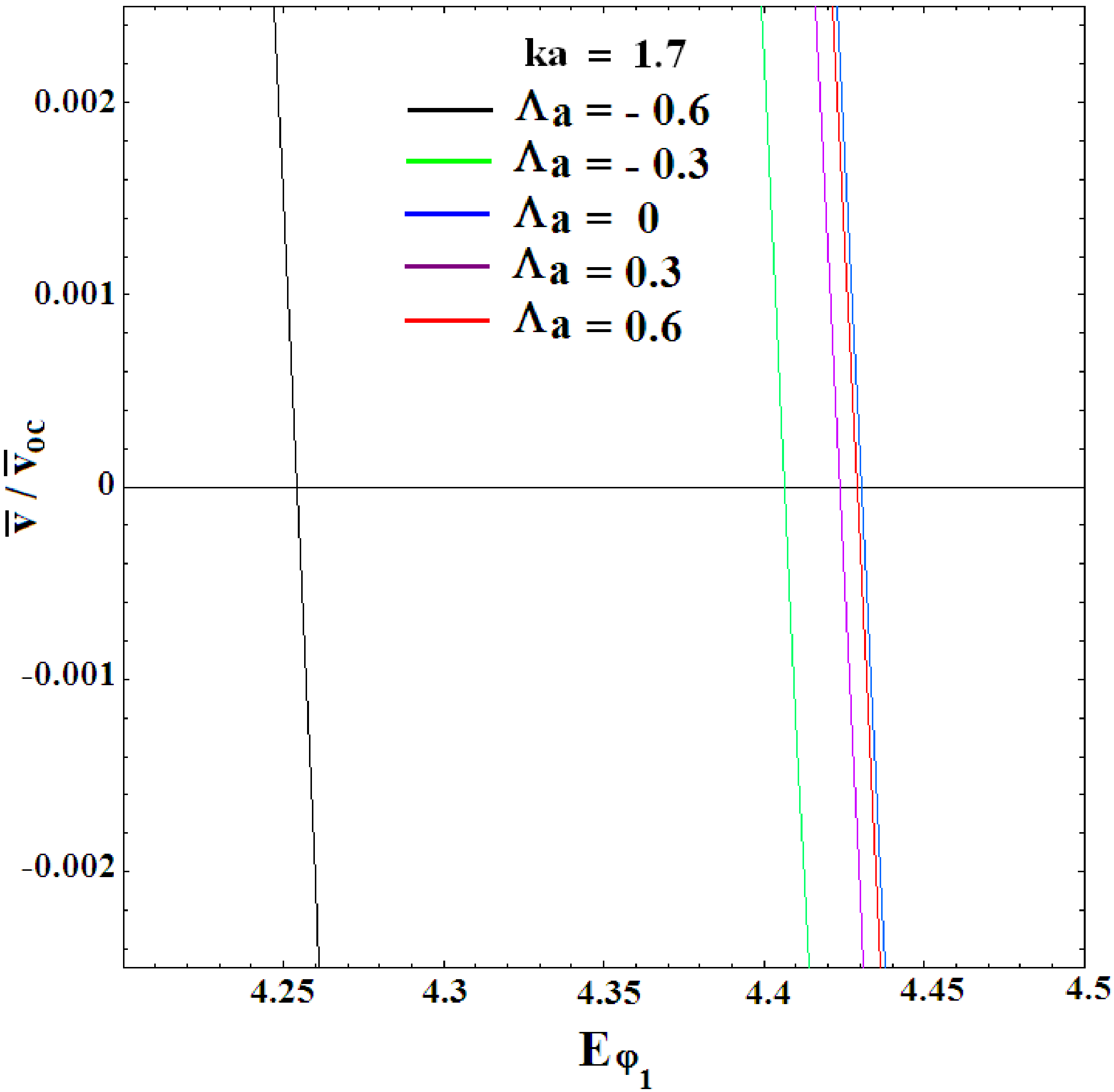
Figures 7(a), (b) and (c) are similar to those in Figures 6(a), (b) and (c), but now with three different triangularities for a given elongation $k(a)=1.7$. Two positive and two negative triangularities are considered. The best value is for $\Lambda(a)=-0.6$ and later for $\Lambda(a)=0.6$

Negative triangularity produces better confinement that the negative ones. There are also better results with the hole case. Increasing values of triangularity, independent of the sign, improve confinement.



ka = 1.7
Λa = - 0.6
Λa = - 0.3
Λa = 0
Λa = 0.3
Λa = 0.6
0.002
0.001
0
-0.001
-0.002
4.6
4.7
4.8
4.9
$\bar{v} / \bar{v}_{0c}$
E_{φ_1}

Figure 7a



ka = 1.7
Λa = - 0.6
Λa = - 0.3
Λa = 0
Λa = 0.3
Λa = 0.6
0.002
0.001
0
-0.001
-0.002
v̄ / v̄0c
4.25
4.3
4.35
4.4
4.45
4.5
Eφ1

Figure 7b

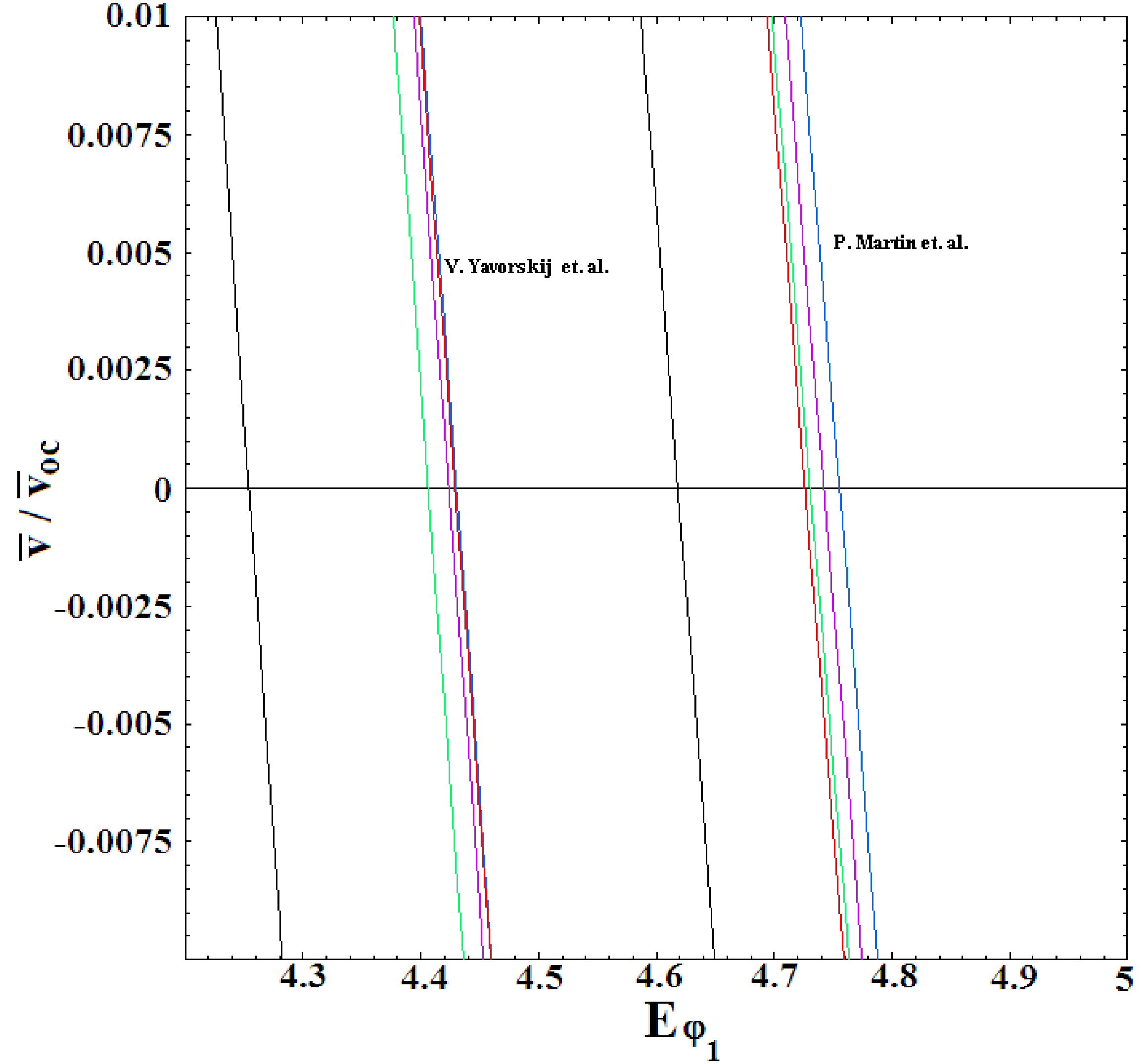

0.01
0.0075
0.005
0.0025
0
-0.0025
-0.005
-0.0075
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
$\overline{V} / \overline{V}_{0c}$
E_{φ_1}
V. Yavorskij et. al.
P. Martin et. al.

Figure 7c

Figures 8(a) and (b), show the dimensionless diffusion velocity versus elongation for different inductive dimensionless fields, $\widetilde{E}_{\varphi_1} = 4.7,\ 4.75$, 4.8 and 4.85. The triangularity is the same for three Figures, $\Lambda(a) = 0.28$

Improve confinement is obtained increasing $\widetilde{E}_{\varphi_1}$, and better results are obtained with holes currents.

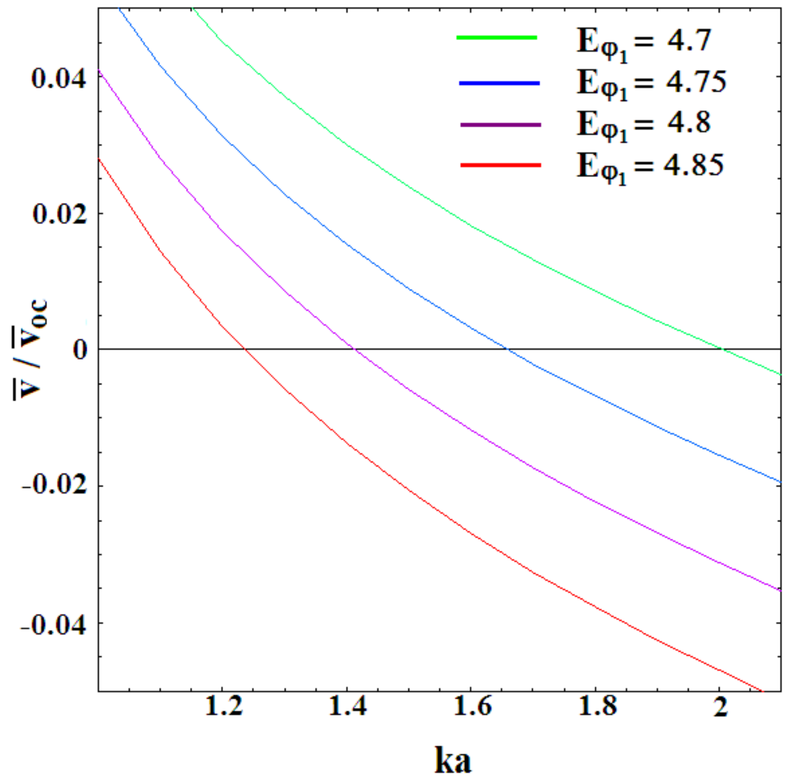
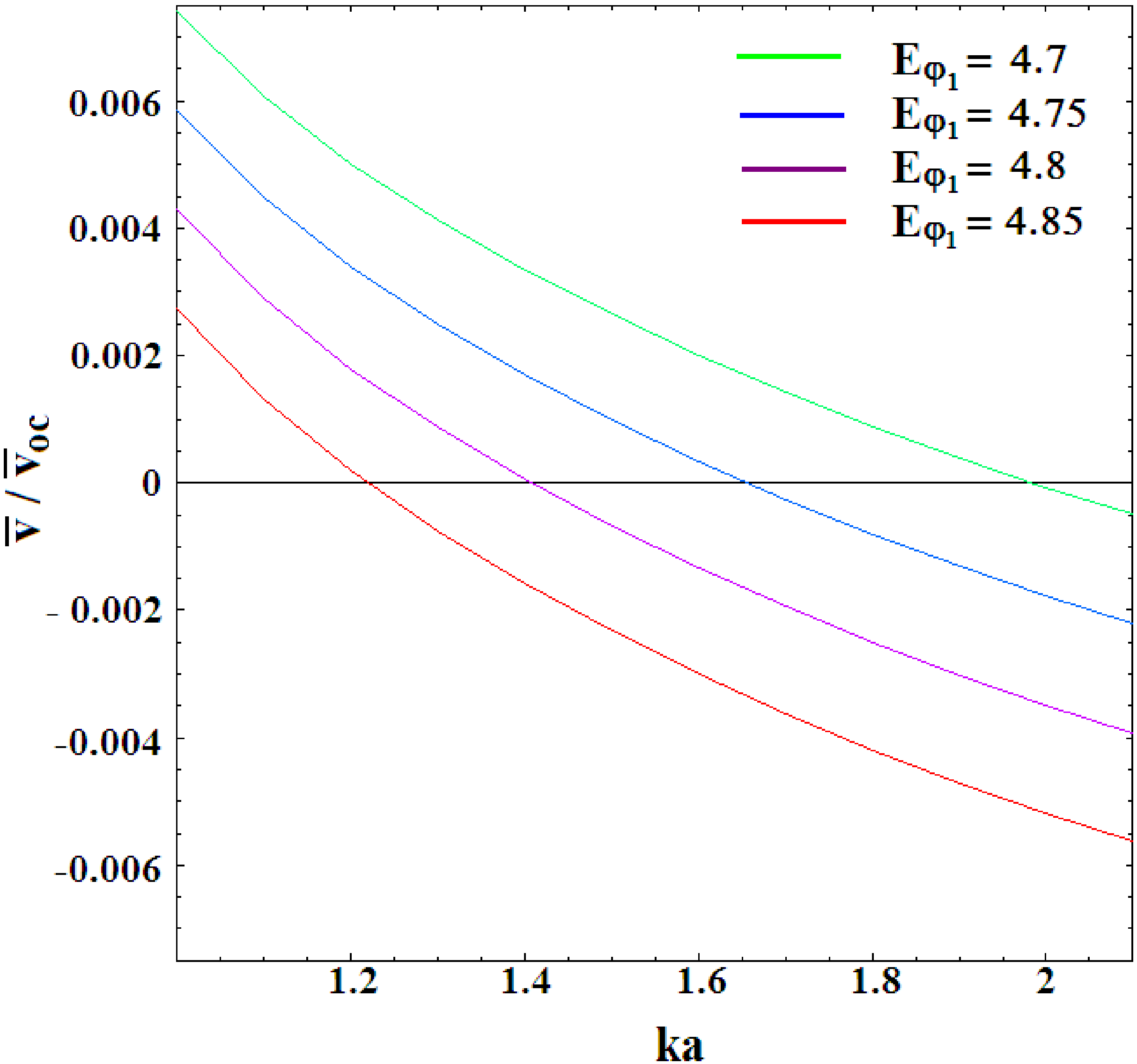


Figure 8a:



$E_{\varphi_1} = 4.7$
$E_{\varphi_1} = 4.75$
$E_{\varphi_1} = 4.8$
$E_{\varphi_1} = 4.85$
0.006
0.004
0.002
0
- 0.002
-0.004
-0.006
$\overline{V} / \overline{V}_{oc}$
1.2
1.4
1.6
1.8
2
ka

Figure 8b:

In Figure 9, the analysis is performed in a different way, here $\widetilde{E}_{\varphi\ \mathrm{marginal}}$ is defined as the inductive electric field producing zero dimensionless diffusion velocity. In this way the confinement zone is separated from the non-confinement region for the curves show in those figures. First the case of changing elongation is considered and later, changing triangularity for the same elongation. As in the previous Figures the hole case leads to better confinement than the no-hole case.

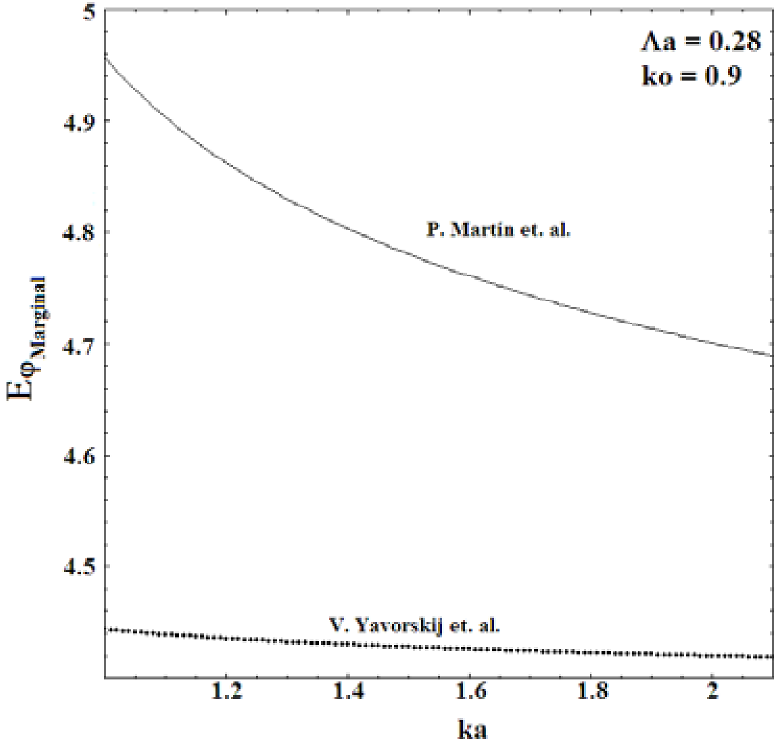


Figure 9:

Figure 10a:

No-hole

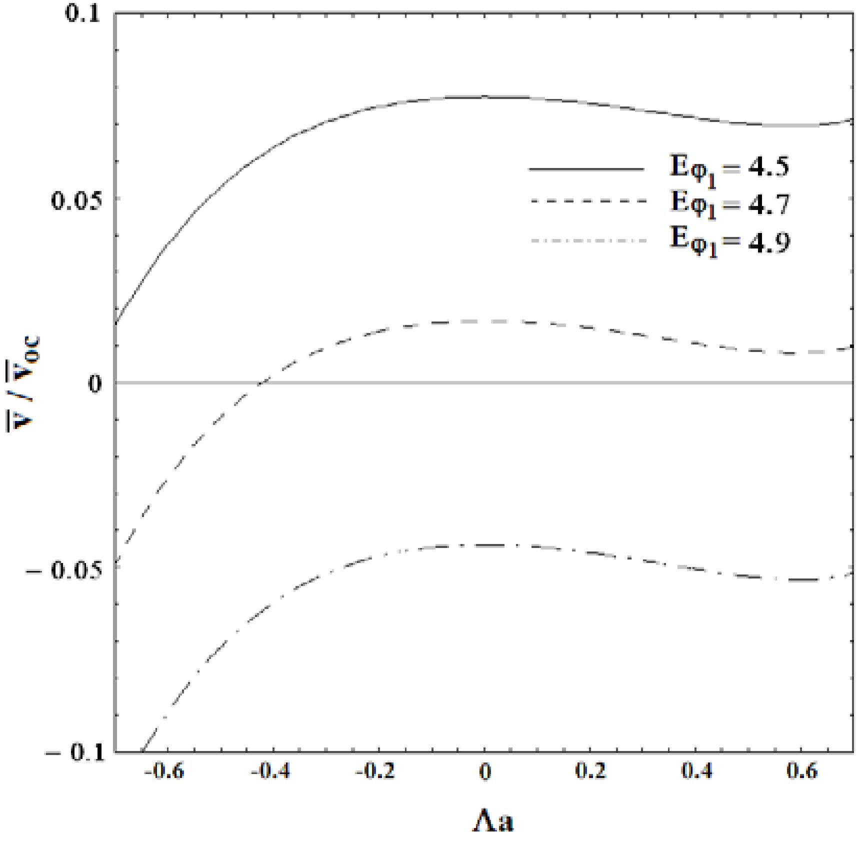
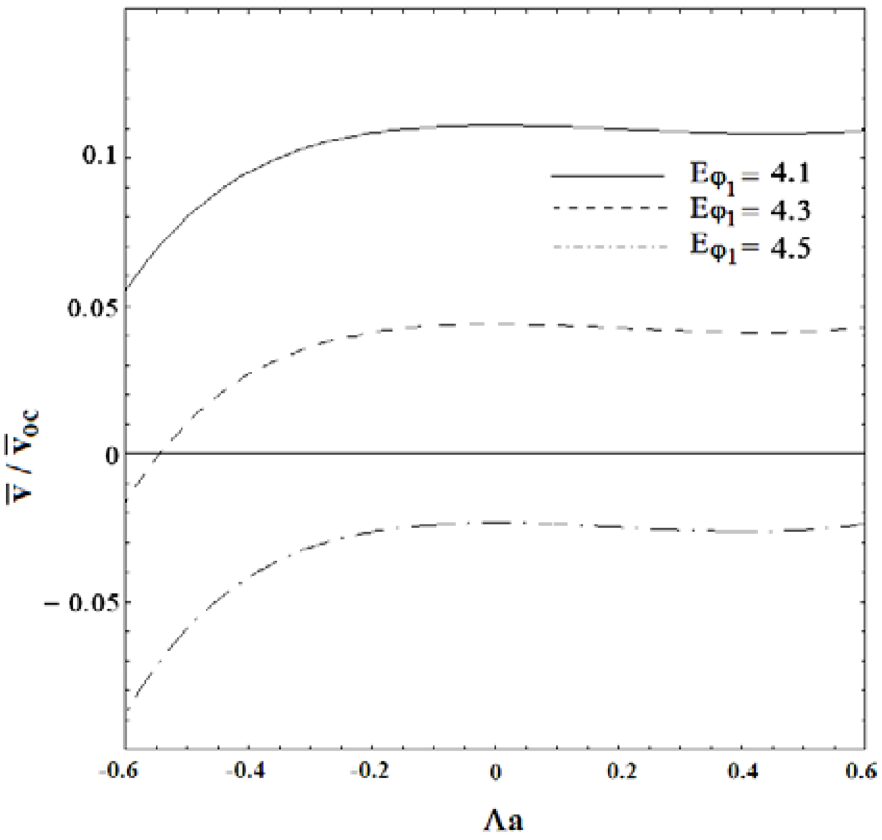
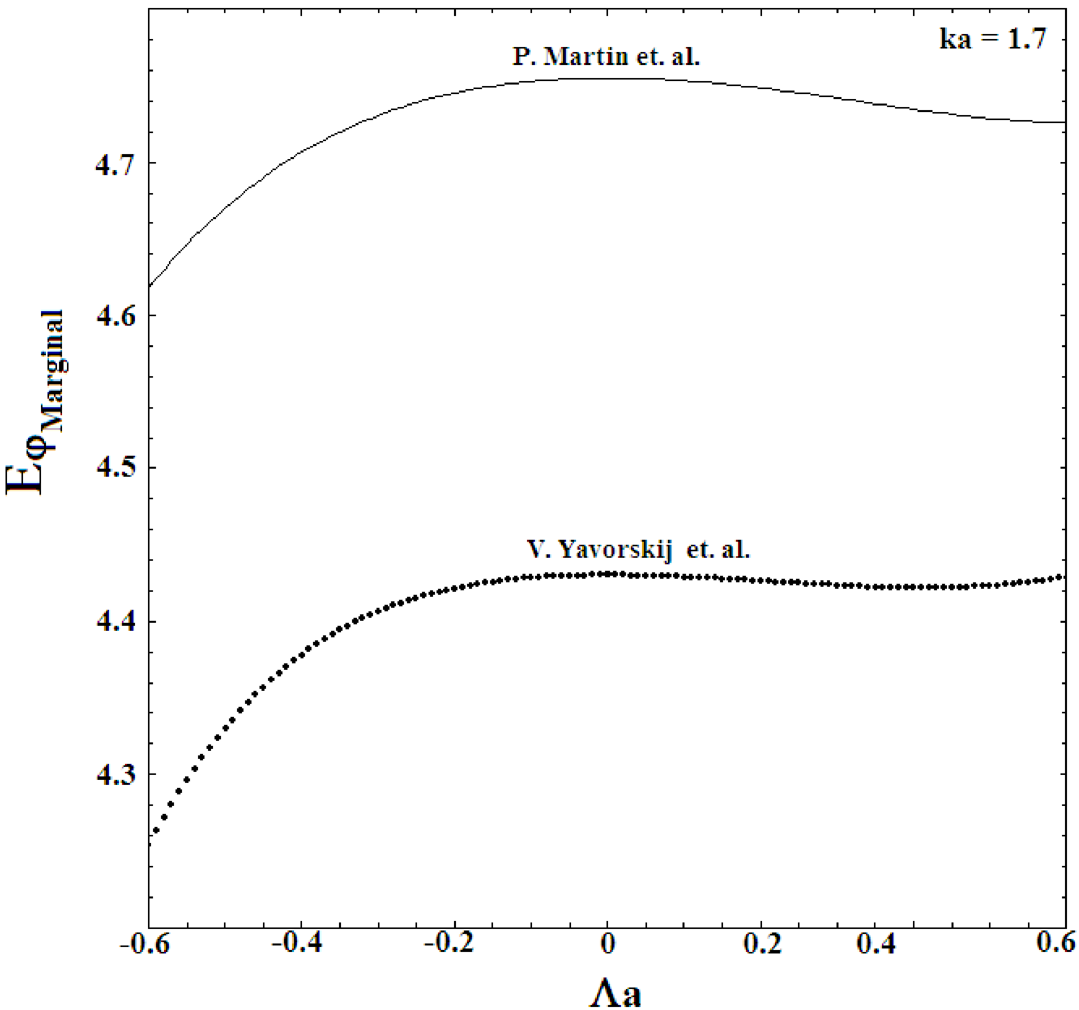


Figure 10b

With hole





ka = 1.7
P. Martin et. al.
V. Yavorskij et. al.
4.7
4.6
4.5
4.4
4.3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Λa
Eφ Marginal

Figure 11:

CONCLUSION

Our analysis shows that confinement is much better in tokamaks where a hole current has been produce than in the cases with no hole .

In general, confinement increases with increasing elongation and triangularity. For the same absolute value of triangularity the confinement is better with negative than positive triangularities. However, for large values of elongation and triangularity there is not increasing confinement, and in these cases a plateau is reached

Triangularity seems to be more important for confinement than elongations.